647510 **BSCgraCC6010** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination
March/April-2023 (OLD COURSE)
GRAPH THEORY AND COMPLEX ANALYSIS -2(CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(A) Answer the following question. (Any One) (07)

1. Define Euler graph. State and prove necessary and sufficient condition for being Euler graph.
2. Prove that in complete graph with n vertices there are $\frac{n-1}{2}$ edge disjoint Hamiltonian circuits, where n is odd no. ≥ 3.

Q.1(B) Answer the following questions. (Any One) (04)

1. Define
   - **I.** Sub graph **II.** Edge-disjoint Sub graphs
   - **III.** closed Walk **IV.** Euler graph.
2. State and prove First theorem of Graph Theory.

Q.1(C) Answer the following question. (Any One) (03)

1. Define: Parallel edges, Regular graph and Euler graph.
2. Prove that in complete graph number of edges is $\frac{n(n-1)}{2}$.

Q.2(A) Answer the following question. (Any One) (07)

1. State and prove Euler's theorem for planner graph.
2. In usual notation for any simple, connected planer graph prove that $e \geq \frac{3}{2}f$ and $e \leq 3n-6$.

Q.2(B) Answer the following questions. (Any One) (04)

1. Define planner graph. Prove that complete graph with 5 vertices is non planner.
2. Prove that a vertex v in a connected graph G is cut vertex if and only if there exist two vertices x and y in G such that every path between x and y passes through v.

Q.2(C) Answer the following question. (Any One) (03)

1. Define proper colouring of graph, Chromatic number.
2. Define Path matrix with example.

Q.3(A) Answer the following question. (Any One) (07)

1. In usual notation prove that the bilinear transformation maps three given distinct points z_1, z_2 and z_3 onto three specified distinct points w_1, w_2 and w_3 is $\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)} = \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$.
2. Discuss the bilinear transformation $W = \frac{1}{z}$ $z \neq 0$.

Q.3(B) Answer the following questions. (Any One) **(04)**

1. Show that the tangent at $P(z_0)$ to the curve C in z-plane is rotated an angle φ_0=Arg $f'(z_0)$ for the corresponding curve C' in w-plane under the transformation w = f(z).
2. Prove that every Mobious transformation maps circles OR straight lines into circles OR straight lines.

Q.3(C) Answer the following question. (Any One) **(03)**

1. Prove that the transformation $w = \frac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ of the z-plane into a line 4u+3=0 in w-plane.
2. Obtain a transformation of sector $r \leq 1,\ 0 \leq \theta \leq \frac{\pi}{4}$ under the transformation $w = z^2$.

Q.4(A) Answer the following question. (Any One) **(07)**

1. State and prove Taylor's theorem for an analytic function.
2. State and prove Laurent's theorem for an analytic function.

Q.4(B) Answer the following questions. (Any One) **(04)**

1. Expand $\frac{1}{z}$ in increasing power of (z-1).
2. Expand $\frac{z}{(z-1)(z-3)}$ in Laurent's series for $0 < |z-1| < 2$.

Q.4(C) Answer the following question. (Any One) **(03)**

1. Find the region of convergence and radius of convergence for $\sum_1^{\infty} \frac{z^n}{7^n+1}$.
2. Prove: $\cos z = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{(2n)!}$.

Q.5(A) Answer the following question. (Any One) **(07)**

1. Prove by using Residue Theorem $\int_0^{\infty} \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{4a^2}$; a > 0.
2. Prove by using Residue Theorem $\int_0^{\pi} \frac{d\theta}{(2+\cos\theta)^2} = \frac{2\pi}{3\sqrt{3}}$.

Q.5(B) Answer the following questions. (Any One) **(04)**

1. State and prove Cauchy's residue theorem.
2. Evaluate $\int_C \frac{5z-2}{z(z-1)}\,dz$; C: $|z| = 2$.

Q.5(C) Answer the following question. (Any One) **(03)**

1. Discuss the method for finding the residue of f(z) at the z_0 of the order m; $m \geq 2$.
2. Find: $\text{Res}\left(\frac{e^{2z}}{(z-1)^2}, 1\right)$

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